Intercomparison of Millimeter-Wave Radiative Transfer Models

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***Abstract*—This study analyzes the performance at millimeter-wave frequencies of five radiative transfer models, i.e., the Eddington second-order approximation with and without δ-scaling, the Neumann iterative method with and without geometric series approximation, and the Monte Carlo method. Three winter time precipitation profiles are employed. The brightness temperatures calculated by the Monte Carlo method, which considers all scattering angles, are considered as benchmarks in this study. Brightness temperature differences generated by the other models and sources of those differences are examined. In addition, computation speeds of the radiative transfer calculations are also compared. Results show that the required number of quadrature angles to generate brightness temperatures consistent with the Monte Carlo method within 0.5 K varies between two and six. At least second to 15th orders of multiple scattering, depending on the significance of scattering, are required for the Neumann iterative method to represent accurately the inhomogeneous vertical structure of the scattering and absorbing components of precipitating clouds at millimeter-wave frequencies. The δ-scaling in the Eddington second-order approximation improves brightness temperatures significantly at nadir for cloud profiles that contain snow due to the correction for strong scattering, while it did not make any difference at 53° off-nadir. The computational time comparisons show that the Neumann iterative method generates accurate brightness temperatures with better computational efficiency than the Monte Carlo method for cloud profiles with weak scattering. However, it can consume computational time that is even greater than the Monte Carlo method for some millimeter-wave frequencies and cloud profiles with strong scattering. A geometric series approximation can improve computational efficiency of the Neumann iterative method for those profiles. In view of the ease of introducing scaled parameters into the Eddington second-order approximation, good computational time efficiency, and better than within 2 K accuracy when compared with the Monte Carlo method, we recommend its use for brightness temperature calculations at millimeter-waves in precipitating atmospheres.**

***Index Terms*—Delta-scaling, Eddington second-order approximation, millimeter-wave, Monte Carlo method, Neumann iterative method, radiative transfer model.**

I. Introduction

ACCURATE and computationally efficient forward radiative transfer calculations are essential for the operational retrieval of atmospheric properties from remotely sensed microwave observations. Interest has grown in the retrieval of hydrometeor parameters such as mixing ratio and density from radiometers operating at millimeter microwave frequencies that were originally selected for temperature and moisture sounding. The Special Sensor Microwave/Temperature-2 (SSM/T-2) [1] on the Defense Meteorological Satellite Program (DMSP) and the Advanced Microwave Sounding Units (AMSU-A and B) [2] on the National Oceanic and Atmospheric Administration (NOAA) 15, 16, and 17 satellites provide such measurements. One objective of the Global Precipitation Measurement (GPM) mission [3], which is a follow-on, multisatellite extension of the Tropical Rainfall Measuring Mission (TRMM) [4], is to measure precipitation at high latitudes where a significant portion of total precipitation is frozen. It is expected that at least some of the GPM constellation satellites will have radiometers with millimeter-wave channels.

In order to develop fast inversion algorithms, pregenerated brightness temperatures ($T_{\rm b}$) are employed as a lookup table. To make a retrieval algorithm applicable to various precipitation systems, many atmospheric and hydrometeor profiles are required in a lookup table. This requires radiative transfer models that are fast, yet accurate.

Kummerow [5] examined the ability of the Eddington second-order (E2O) approximation to properly capture the angular distribution of the radiation by comparing eight-stream discrete ordinate solutions at frequencies between 6.6–183 GHz using a simplified three-layer cloud model. Smith *et al.* [6] compared radiative transfer models used in generating databases for satellite rainfall retrieval algorithms at frequencies of the TRMM Microwave Imager (TMI) channels (10.7–85.5 GHz). These models included the E2O approximation, 16-stream discrete ordinate method, and the Monte Carlo (MC) method. They compared the resulting $T_{\rm b}$'s calculated with four different rainfall profiles. The purpose of their study was to ensure that differences obtained from retrieval techniques do not originate from the underlying radiative transfer code employed for the forward modeling of rain profiles.

This paper, which addresses the accuracy of various radiative transfer models in precipitating clouds, expands the intercomparisons of Smith *et al.* [6] to higher frequencies (millimeter-wavelengths) where absorption from dry air and water vapor and scattering from ice particles become significant. The performance of five radiative transfer models, the E2O approximations [7], [8] with (DE2O) and without δ-scaling [9], the Neumann iterative (NI) method [10], [11] with and without the geometric series (GS) approximation [12], and the MC method [13] are compared at frequencies of 89, 118.87, 150, 183.3±1, 183.3±3, 183.3±7, 220, 340 GHz at nadir and at 53° off-nadir. Three winter time at-

Manuscript received September 22, 2003; revised May 28, 2004. This work was supported by the National Aeronautics and Space Administration under Grants NCC-5-584 and S-69019-G.

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Digital Object Identifier 10.1109/TGRS.2004.833392

mospheric profiles (rain over the ocean surface, rain plus snow over the ocean surface, and heavy dry snow over a land surface) with 35 layers are displayed for these comparisons. Since these high frequencies are sensitive to ice scattering when compared with the TMI channels employed by Smith *et al.* [6], this study examines the multiple-scattering effect in radiative transfer calculations. In addition, we seek to identify the differences and source of those differences in T_{b}'s that originate in the forward computations. The relative comparisons of computational time depending on profile and frequency are also presented.

II. Radiative Transfer Models

A. Eddington Second-Order Approximation

Representation of microwave radiances from horizontally homogeneous clouds of hydrometeors can be calculated analytically by representing angular dependence of the radiances and phase functions by a linear polynomial in cosine of the zenith and scattering angles, respectively [7], [8]. Many studies [6], [9], [14] showed that the E2O approximations [14] were reasonably accurate at frequencies less than 85 GHz with little absorption compared to other radiative transfer models such as discrete ordinate and the MC methods [13].

The E2O approximations assume that the scattered radiation reaches a diffusive equilibrium, i.e., numerous scattering events have occurred so that the scattering source term is a linear function of the cosine of the observing angle, and the thermal source term is isotropic. Because of its calculation efficiency while keeping reasonable accuracy, the radiative transfer model with the E2O approximation has been applied to many passive-microwave remote sensing studies using frequencies up to 85 GHz [6], [15], [16].

B. Eddington Second-Order Approximation With δ-Scaling

This DE2O model is identical to the previously cited the E2O approximation except that the profiles of asymmetry factor, extinction coefficient, and albedo for single scattering are scaled to preserve the second moment of the phase function [9]. The forward scattering by hydrometeors becomes prominent as particle size and frequency increase. The forward scattering peak takes on a resemblance to a Dirac δ-function when plotted as a function of the cosine of the scattering angle, while the remainder of the phase function is expanded as a linear function of the cosine of the scattering angle [9].

C. Neumann Iterative Method

The underlying principles of the Neumann series, or iterative, solution of the radiative transfer equation are described in [10] who applied the technique to compute radiances emerging from a cloud illuminated by solar radiation. The application of the NI series expansion to microwave radiative transfer in cloudy or precipitating atmospheres has been well described in [11]. The NI solution is the natural extension of radiative transfer models that describe purely absorbing atmospheres in which scattering is considered as a perturbation. Once the unscattered radiance is computed, that radiance can be introduced into the source term describing single scattering, and the process can be repeated to derive successive orders of scattering.

V. Conclusion

The E2O and DE2O approximations, the NI method with and without the GS approximation, and the MC method to calculate T_{b}'s are compared in terms of the accuracy and the computational efficiency at millimeter-wave frequencies. The T_{b}'s calculated by the MC method, which considers all scattering angles, are considered as benchmarks in this study. Three winter time atmospheric profiles (rain over the ocean surface, rain plus snow over the ocean surface, and heavy dry snow over a land surface) with 35 layers are employed for these comparisons.

1) The appropriate number of quadrature angles varies between two and six, and at least second to 15th orders of multiple scattering, depending on the significance of scattering, are required for the NI method to represent accurately T_{b}'s from inhomogeneous vertical structure of the scattering and absorbing components of precipitating clouds.
2) An insufficient number of quadrature angles in the NI method can cause T_{b} biases up to 12 K at millimeter-wave frequencies for hydrometeor profiles with heavy snow.
3) The DE2O approximation improved T_{b}'s significantly at nadir for cloud profiles including frozen hydrometeors due to the correction for strong scattering, while it did not make any difference at 53° off-nadir.
4) The computation time comparisons (Table IV) show that the NI method can consume computer time that is even greater than the MC method at millimeter-wave frequencies for profiles with strong scattering caused by frozen hydrometeors. However, the computational time of the NI method for these profiles can be improved significantly by using the GS approximation.

Despite the fact that the iterative T_{b}'s are generally closer in value to the MC T_{b}'s, the iterative accuracy advantage requires significant preprocessing in order to determine the required number of quadrature angles. If the number of quadrature angles is selected at a high number to avoid preprocessing, the computational time suffers. The MC method is also computationally inefficient with respect to the E2O model. In view of the ease of introducing scaled parameters into the E2O approximation with δ-scaling, good computational time efficiency, and better than 2 K accuracy, we recommend its use for millimeter-wave radiative transfer calculations in winter time precipitating atmosphere.