A Broadband Microwave Radiometer Technique at X-band for Rain and Drop Size Distribution Estimation

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***Abstract*—Radiometric brightness temperatures below about 12 GHz provide accurate estimates of path attenuation through precipitation and cloud water. Multiple brightness temperature measurements at X-band frequencies can be used to estimate rainfall rate and parameters of the drop size distribution once correction for cloud water attenuation is made. Employing a stratiform storm model, calculations of the brightness temperatures at 9.5, 10, and 12 GHz are used to simulate estimates of path-averaged median mass diameter, number concentration, and rainfall rate. The results indicate that reasonably accurate estimates of rainfall rate and information on the drop size distribution can be derived over ocean under low to moderate wind speed conditions.**

***Index Terms*—Airborne radiometer, drop size distribution, microwave radiometry, rain rate estimation, spaceborne radiometer.**

I. Introduction

For frequencies up to about 12 GHz, the radiometric brightness temperature from rain over ocean is closely related to the path integrated attenuation (PIA) for low to moderate surface wind speeds. Multifrequency measurements of brightness temperature at X-band (8.2–12.4 GHz) over a bandwidth of 2–3 GHz, in principle, provide sufficient information to estimate parameters of the drop size distribution and rainfall rate. The purpose of this paper is to derive equations for and investigate the feasibility of this type of parameter estimation.

The instrument concept is similar to that used in the stepped-frequency microwave radiometer (SFMR). In the SFMR, multiple bands in the frequency range from 4.6–7.2 GHz are used to estimate rainfall rate and near-surface wind speed over the ocean [1], [2]. Here, the emphasis is different in the sense that the objective is to estimate parameters of the path-averaged drop size distribution (DSD) and rain rate using X-band frequencies. The approach considered here is also related to that used for estimation of DSD parameters from multifrequency transmission measurements along a microwave link [3]–[5]. In essence, two path-attenuation measurements yield two parameters of the exponential form of the DSD or, equivalently, two parameters of the gamma DSD with the shape parameter, μ, fixed or expressed as a function of one of the variable parameters [6]. There are, however, several complicating factors in the application of this technique to microwave radiometry. The first is that conversion of brightness temperature T_B to path-integrated attenuation A is not one-to-one: changes in the characteristics of the mixed-phase particles and scattering contributions from the ice and water introduce uncertainty in the conversion of T_B to A that translate into estimation errors. Cloud liquid water presents a somewhat different problem. Cloud droplets are Rayleigh scatterers/absorbers within the frequency band of interest so that the functional dependence of the specific attenuation on frequency is known. This contribution can be eliminated in part by considering the difference of path attenuations with suitable normalizations. Once the rain parameters are estimated from differential quantities, the cloud liquid water, in principle, can be recovered from the equation for total path attenuation.

For operation below or above X-band, the error sources become larger. Below X-band, the differential attenuation is typically small and the relative errors in the estimate render the estimates inaccurate. While the differential attenuation is relatively strong above 12 GHz, greater scattering contributions to T_B from ice and snow, as well as the rain itself, introduce an increasing amount of variability into the T_B–A relationship, making the estimation of DSD parameters impractical.

The use of closely spaced frequencies has also been considered for airborne and spaceborne weather radars [7], [8]. For radar, the differential reflectivity factor serves as an estimator of the median mass diameter, D_0, of rain as well as snow. For the X-band radiometer, a ratio of normalized differential path attenuations, derived from brightness temperatures, provides an estimator for the path-averaged median mass diameter of the rain. In a sense, the radiometer-based algorithm is more straightforward: for the radar application, correction for attenuation must be made before the differential reflectivity can be used whereas for the radiometer algorithm, a function of the differential attenuation serves as a direct estimator of the path-averaged D_0 in rain. As in most applications of air- or spaceborne radar and radiometer to precipitation, the primary advantage of the radar is its range-profiling capability while the attractions of the microwave radiometer are higher reliability, lower cost, and, typically, more rapid scanning capabilities.

II. Algorithm Considerations

The first objective is to express parameters of the DSD and rain rate as functions of the PIA or differential PIA. In the following section, these quantities are related to the brightness temperatures so that a connection is made between the measurements and the quantities to be estimated.

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Let $\widetilde{A}(f)$ be the PIA (decibels) at frequency f (hertz) from the storm top (range $r = 0$) to the surface ($r = r_s$), and let $k(f, s)$ be the specific attenuation (dB $\cdot$ km^{-1}) at $r = s$. The quantities $\widetilde{A}(f)$ and $k(f, s)$ are related by

$$\widetilde{A}(f) = \int_0^{r_s} k(f, s) ds. \quad (1)$$

The specific attenuation consists of terms corresponding to contributions from rain, snow, mixed-phase precipitation, as well as cloud liquid and cloud ice and various atmospheric gases such as water vapor and oxygen. Although the effects of atmospheric gases will be assessed in the error analysis given later, for the purpose of constructing estimators of the DSD parameters, we assume that the contributions from rain and mixed-phase precipitation k_p and that from cloud liquid water k_c dominate so that

$$k = k_c + k_p. \quad (2)$$

As the cloud water droplets much smaller than the wavelengths of interest, k_c can be related to the cloud water content M_c (grams per cubic meter) by [9]

$$k_c = \left[\frac{(0.4343 \times 6\pi f)}{c}\right] \mathrm{Im}(-K) M_c \quad (3)$$

where c is the speed of light (centimeters per second) and K is the dielectric factor related to the complex index of refraction of water m by

$$K = \frac{m^2 - 1}{m^2 + 2}. \quad (4)$$

The imaginary part of the dielectric factor $\mathrm{Im}(-K)$ is a function of frequency and temperature T or height. To remove, in an approximate sense, the influence of cloud water on the DSD and rain rate estimates, consider the following two normalization factors:

$$\alpha_1(f) = \left[\frac{(0.4343 \times 6\pi f)}{c}\right] \quad (5a)$$

$$\alpha_2(f, \overline{T}) = \left[\frac{(0.4343 \times 6\pi f)}{c}\right] \mathrm{Im}(-K(f, \overline{T})) \quad (5b)$$

where $\overline{T}$ in (5b) is the estimated mean temperature of the cloud droplets. Normalizing (3) by α_n gives

$$\frac{k_c}{\alpha_n} = q_n M_c \quad (6)$$

where $n = 1$ or 2 and where

$$q_1 = \mathrm{Im}(-K(f, T)) \quad (7a)$$

$$q_2 = \left[\frac{\mathrm{Im}(-K(f, T)}{\mathrm{Im}(-K(f, \overline{T}))}\right]. \quad (7b)$$

Dividing $\widetilde{A}(f)$ by α_n and letting $\widetilde{A}(f)/\alpha_n = A_n(f)$ then, on using (1)–(3), the normalized path attenuation can be written

$$A_n(f) = \int_0^{r_s} [\alpha_n^{-1}(f, \overline{T}) k_p(f, s) + q_n(f, T, \overline{T}) M_c(s)] ds. \quad (8)$$

If brightness temperature measurements are available at frequencies f_j, f_i ($f_j > f_i$) then the difference of the normalized path attenuations can be written

$$\delta A_n(f_j, f_i) \equiv \alpha_n^{-1}(f_j, \overline{T}) \widetilde{A}(f_j) - \alpha_n^{-1}(f_i, \overline{T}) \widetilde{A}(f_i). \quad (9)$$

Assuming for either value of n that

$$\int_0^{r_s} [q_n(f_j, T, \overline{T}) - q_n(f_i, T, \overline{T})] M_c(s) ds \approx 0 \quad (10)$$

then an approximation for the normalized differential path attenuation, independent of cloud liquid water, is

$$\delta A_n(f_j, f_i) \cong \int_0^{r_s} [\alpha_n^{-1}(f_j, \overline{T}) k_p(f_j, s) - \alpha_n^{-1}(f_i, \overline{T}) k_p(f_i, s)] ds. \quad (11)$$

To understand how information on the raindrop size distribution can be obtained from the measurements, we write the drop diameter distribution $N(D, s)$ (per cubic meter per millimeter) at $r = s$ as

$$N(D, s) = N_t(s) n(D, s) \quad (12)$$

where N_t is the number concentration (per cubic meter). For the log-normal distribution $n(D, s)$ can be expressed as [10]

$$n_{LN}(D; \eta, \sigma) = \left(\frac{1}{\sqrt{2\pi}\sigma D}\right) \exp\left[\frac{-(\ln D - \eta)^2}{2\sigma^2}\right] \quad (13)$$

and for the Gamma distribution as [11]

$$n_G(D; \mu, \Lambda) = \left[\frac{\Lambda^{(\mu+1)} D^\mu}{\Gamma(\mu + 1)}\right] \exp[-\Lambda D] \quad (14)$$

where, in general, η, σ for the log-normal distribution and Λ, μ for the Gamma distribution are functions of height. In the numerical results presented later, we use the median mass diameter D_0 where [10], [11]

$$\Lambda D_0 = 3.67 + \mu \quad (15)$$

for the Gamma distribution and

$$D_0 = \exp[\eta + 3\sigma^2] \quad (16)$$

for the log-normal distribution.

VI. Summary and Conclusion

Brightness temperature measurements at multiple frequencies within X-band, over a span of 2–3 GHz, may provide information on the path-averaged rainfall rate and parameters of the raindrop size distribution. The potential for this type of estimation arises from the fact that at X-band, brightness temperature is well correlated with path-integrated attenuation and that a difference of path attenuations normalized by frequency is nearly independent of cloud liquid water attenuation. Moreover, a ratio of such differences, derived from brightness temperatures at three X-band frequencies, provides an estimate of the median mass diameter D_0 of the rain. The equations also lead to estimates of the mean number concentration, N_t, and rainfall rate. A simple simulation using a stratiform storm model suggests that reasonably accurate rain rates are possible even though the variability in D_0 and N_t can be high. On the other hand, estimates of integrated cloud water content appear to be unreliable. Although the approach appears to be feasible for low to moderate wind speeds over ocean, more detailed studies are needed on variations in the shape parameter, μ, changes in the drop size distribution with height and the effects of the melting layer. The error budget must also include the measurement error of the brightness temperatures in the context of the system design, measurement requirements and scanning strategy.