Conditions for reversal of ice-stream surface slope

Todd K. Dupont & Richard B. Alley

Department of Geosciences & the EMS Environment Institute

The Pennsylvania State University

September 2003

Supported by NSF OPP Grant 0126187

PENNSTATE

MOTIVATION:

Why look at slope reversals?

- reverse hydraulic gradient $\longrightarrow$ outburst floods
- sticky spots or lakes

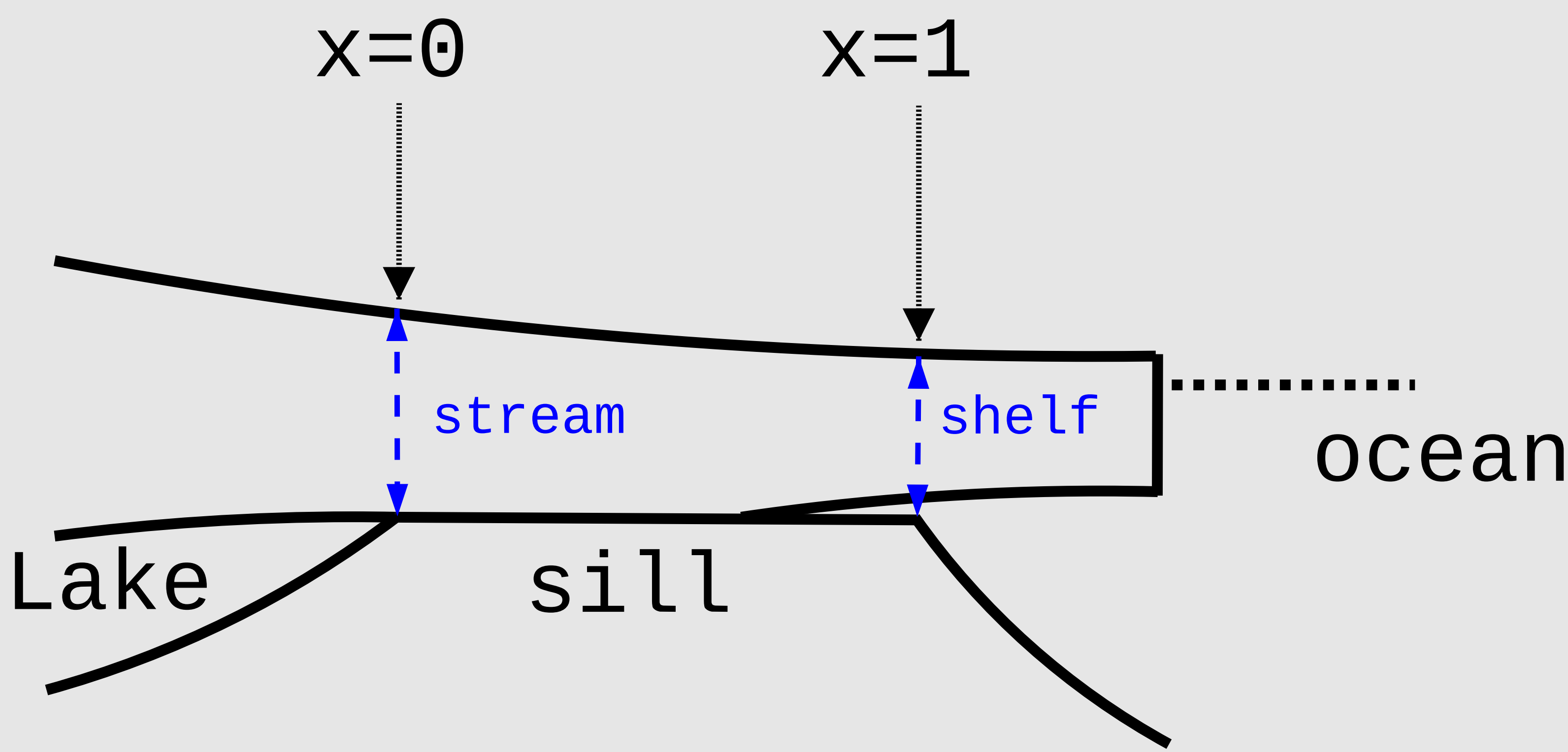
PENNSTATE

QUESTION AND APPROACH

Question: Under what conditions do slope reversals form?

Approach: look for steady-state slope reversals using a simple non-dimensional model

SCHEMATIC GEOMETRY



MODEL

- Momentum balance
  - 1-d version of stream-shelf eqn's - no internal deformation
  - discard lateral drag
- Mass balance
  - from continuity
  - ignore accumulation
  - evolves to steady-state
- Non-dimensional to isolate parameters

MODEL CONT'D: Momentum balance

$$\frac{\partial}{\partial x}\left\{2h\nu\frac{\partial u}{\partial x} - \frac{A_r}{2}h^2\right\} = \begin{cases} G\gamma u & \text{grounded} \\ \frac{A_r}{2}\frac{1}{r_{sw}}h^2 & \text{floating} \end{cases} \qquad x \in (0,1)$$

stretching – press. gradient = basal drag

Upstream boundary condition $u(0) = 1$

Downstream boundary condition

$$\left[2h\nu\frac{\partial u}{\partial x} - \frac{A_r}{2}h^2\right]_{x=1} = f\left(-\frac{A_r}{2}\frac{1}{r_{sw}}h(1)^2\right) + (1-f)\left(-\frac{A_r}{2}h(1)^2\right)$$

- $f = 1 \rightsquigarrow$ a full ice-front condtion; stretching = static pressure difference
- $f = 0 \rightsquigarrow$ no stretching
- $f \sim$ an inverse 'ice shelf buttressing' parameter

PENNSTATE

MODEL CONT'D: Mass balance

$$\frac{\partial h}{\partial t} = 0 = -\frac{\partial (uh)}{\partial x}, \quad x \in (0,1)$$

upstream boundary condition $h(0) = h_0$

- $h_0 \geq 1$
- $h_0 = 1 \rightsquigarrow$ just floating, no overpressure
- $h_0 > 1 \rightsquigarrow$ overpressure

MODEL CONT'D

- ice & basal rheology

$$\nu \equiv |du/dx|^{\frac{1-n}{n}}$$

$$\gamma \equiv |u|^{\frac{1-m}{m}}$$

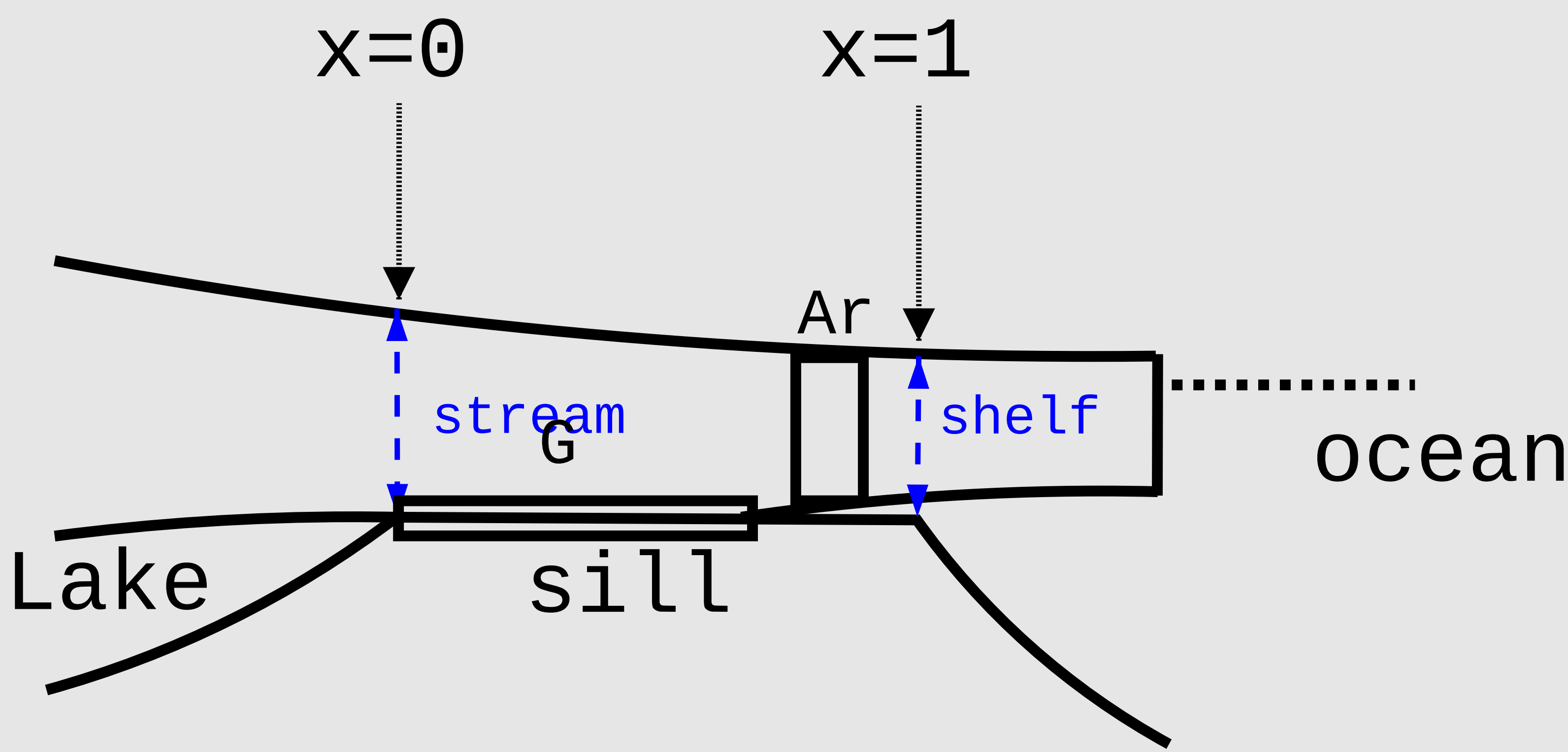
- two key non-dimensional param's

$$A_r = \frac{\textbf{driving stress}}{\text{long. deformation}}$$

$$G = \frac{\textbf{basal drag}}{\text{long. deformation}}$$

PENNSTATE

A_r and G



SCALES AND PARAMETERS

Variable	Scale	Value or range of scale Assumed here
h	$r_{sw}D$	$1.12(1-5\times 10^2)$ m
x	L	$3\times 10^3 - 3\times 10^4$ m
u	U	$2-6\times 10^{-5}$ ms^{-1}
t	L/U	$5-150\times 10^7$ s
B_i	B_i	$1-2\times 10^8$ Pa s$^{1/3}$
τ_b	τ_b	$5\times 10^3 - 2\times 10^5$ Pa
A_r	$\frac{\rho_i g r_{sw} D L^{1/3}}{B_i U^{\frac{1}{3}}}$	$2-60$
G	$\frac{\tau_b L^{4/3}}{r_{sw} D B_i U^{1/3}}$	$5\times 10^{-2} - 6\times 10^2$

EXPERIMENTAL APPROACH

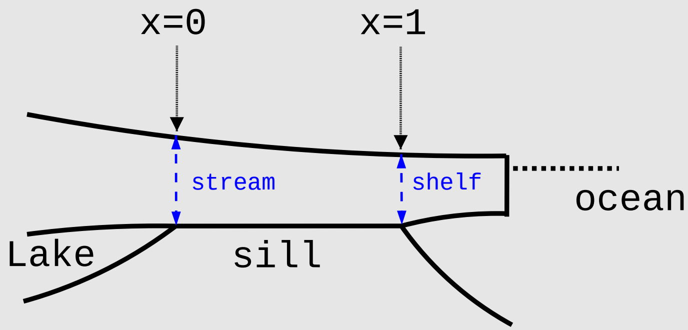
- We're looking for steady-state slope reversals in parameter space.
- 'Free' parameters are A_r, G, h_0 and f.
- Numerical model:
  - FEM spatial discretization of both the mom. and mass balance eqn's
  - FDM temporal discretization of the mass balance eqn
- Relax the model to to equilibrium for each point in parameter space.
- Does the steady-state thickness profile have a slope-reversal at that point in parameter space?

SPECIFIC EXPERIMENTS

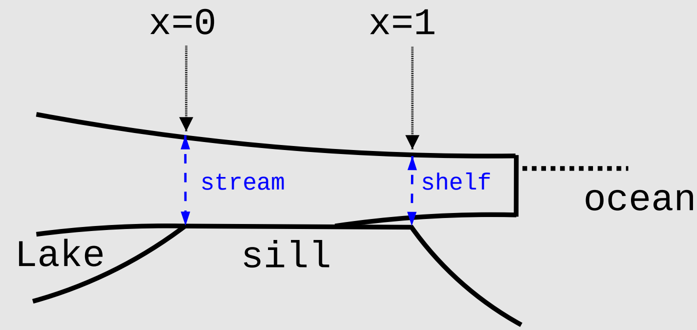
- $2 \leq A_r \leq 30$
- $0.1 \leq G \leq 6$
- $h_0 = \{1, 1.01, 1.1\}$
- $f = \{0.9, 1\}$

PENNSTATE

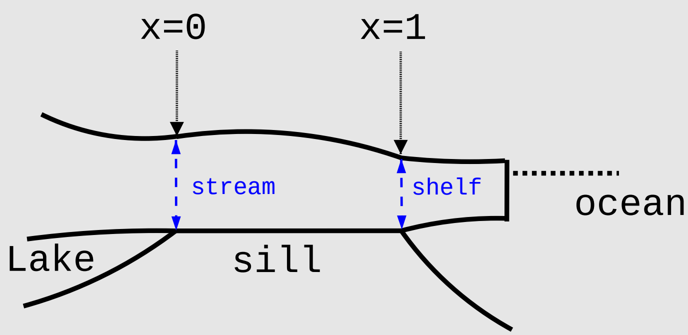
CATEGORIES



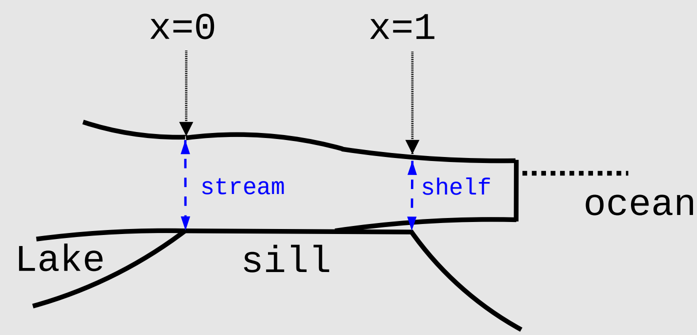
normal slope, fully grounded



normal slope, part. grounded

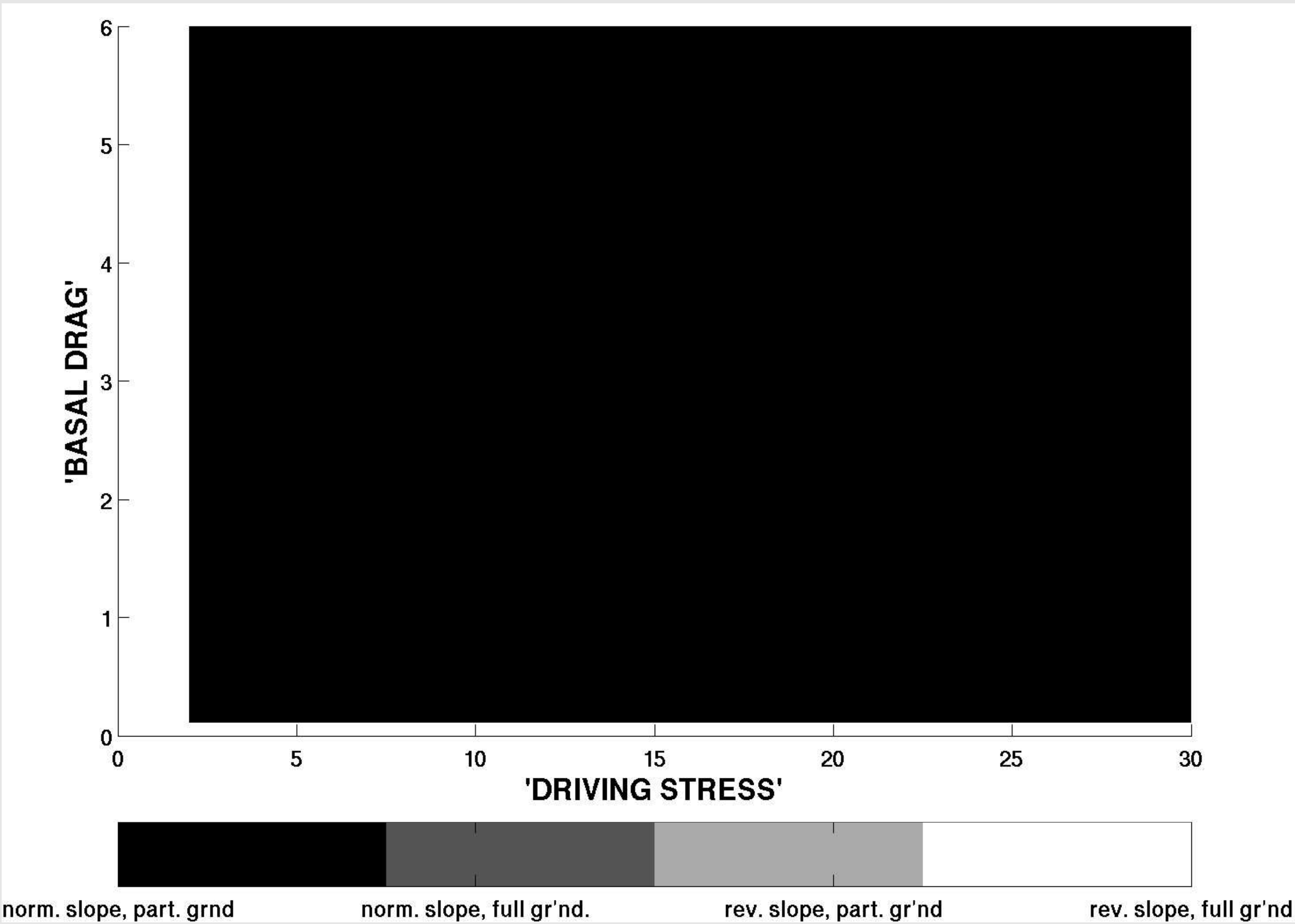


reverse slope, fully grounded

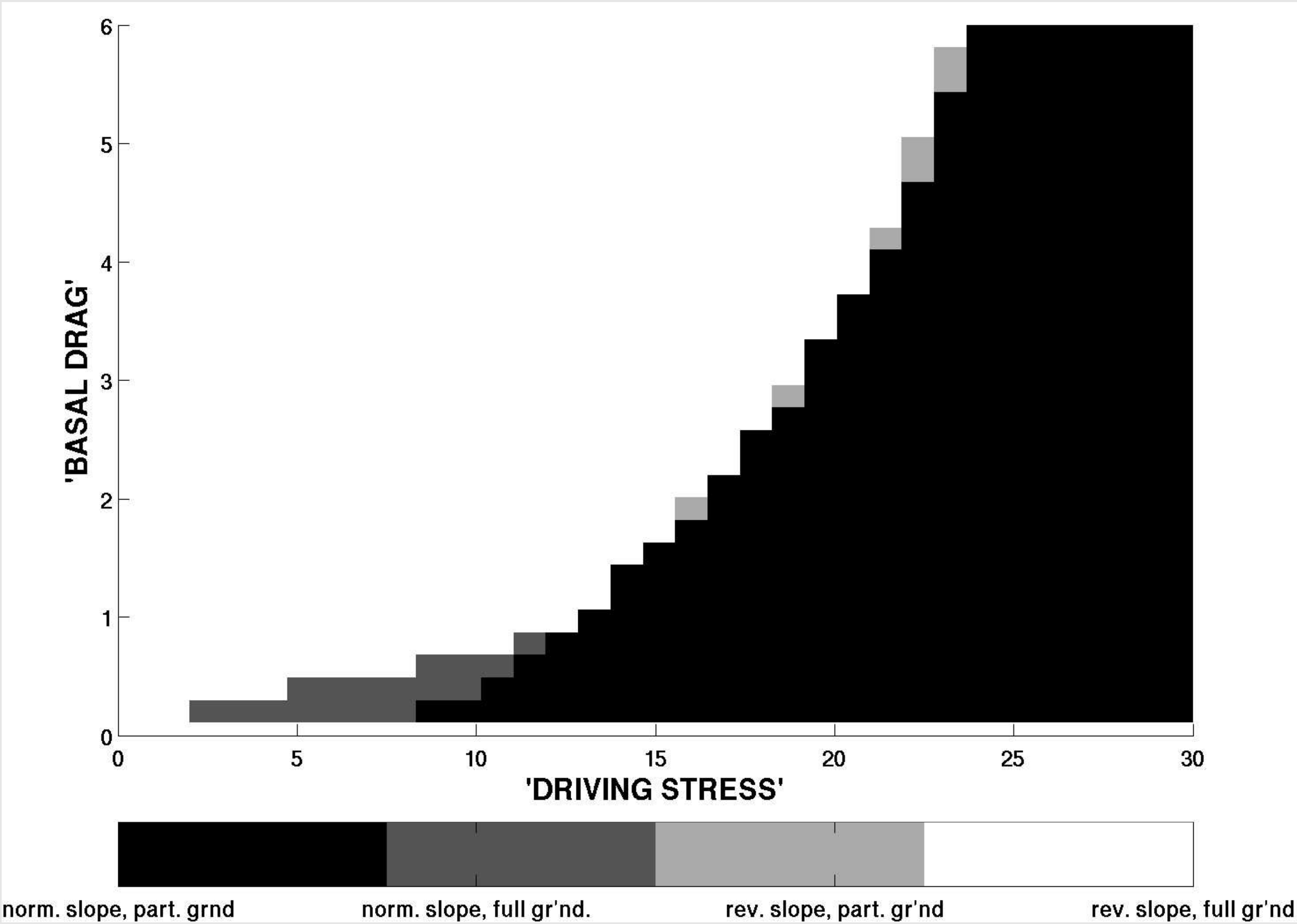


reverse slope, part. grounded

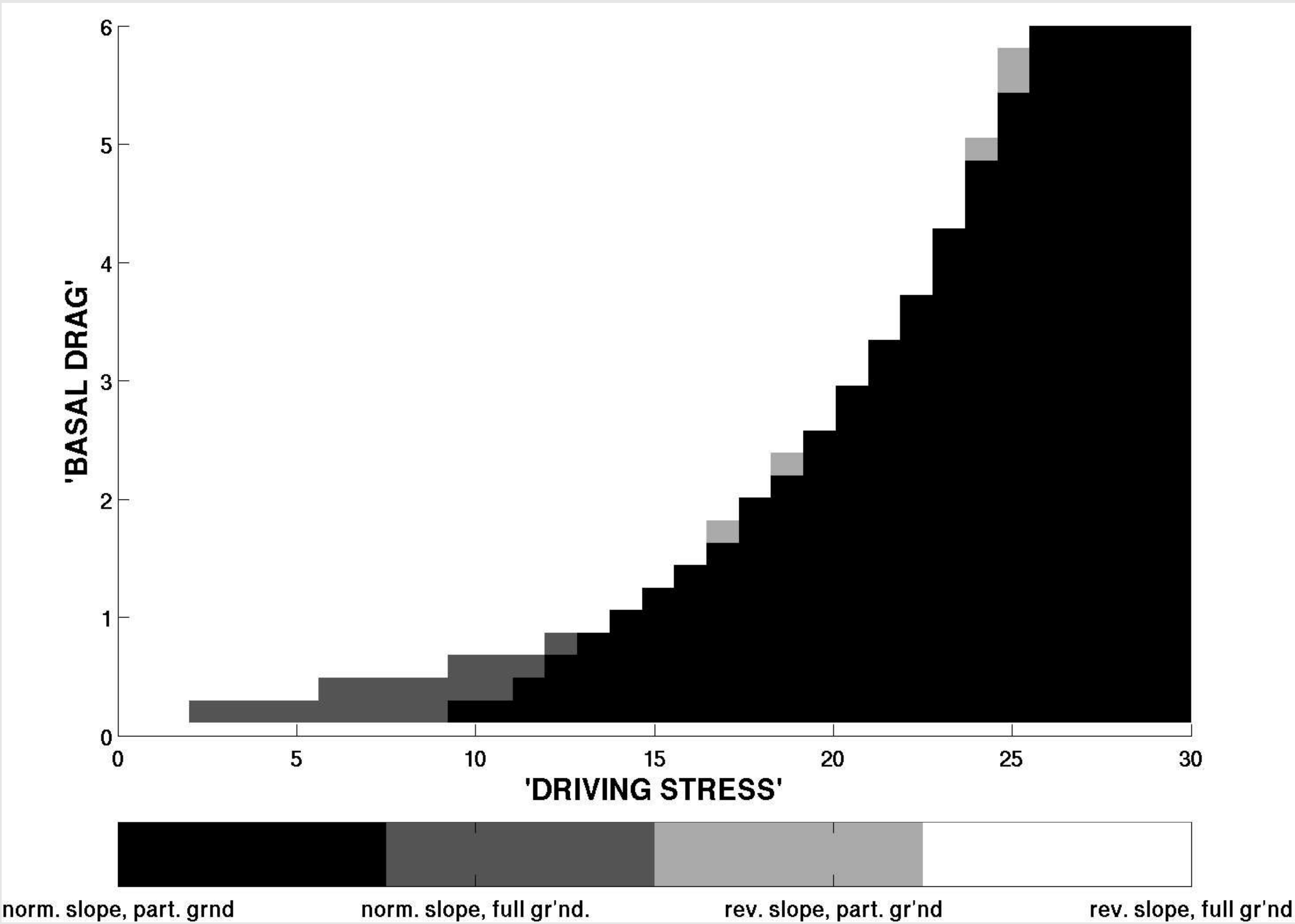
RESULTS: $h_0 = 1$


6
5
4
3
2
1
0
'BASAL DRAG'
0
5
10
15
20
25
30
'DRIVING STRESS'
norm. slope, part. grnd
norm. slope, full gr'nd.
rev. slope, part. gr'nd
rev. slope, full gr'nd

RESULTS: $h_0 = 1.01$, $f = 1$

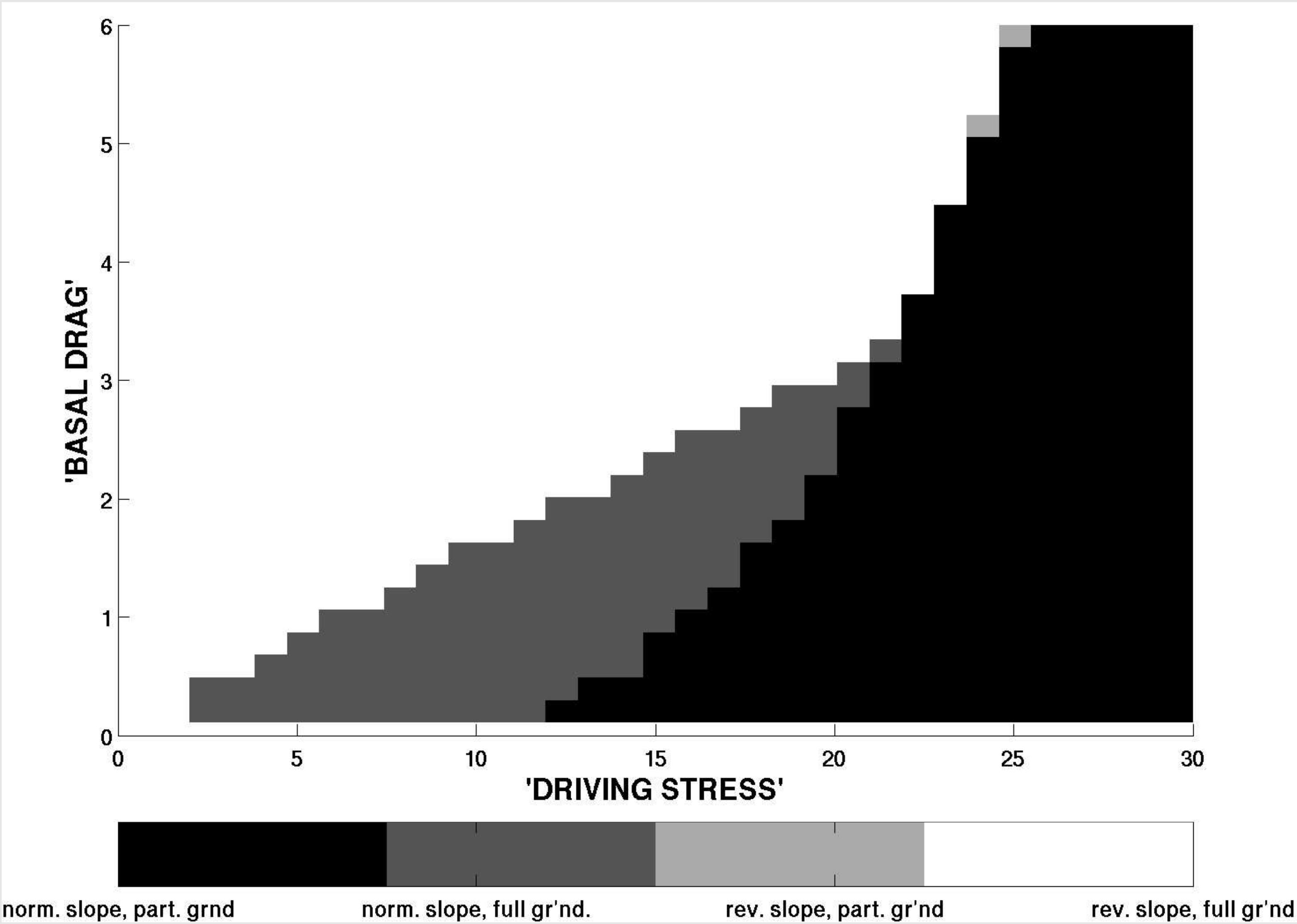


RESULTS: $h_0 = 1.01$, $f = 0.9$

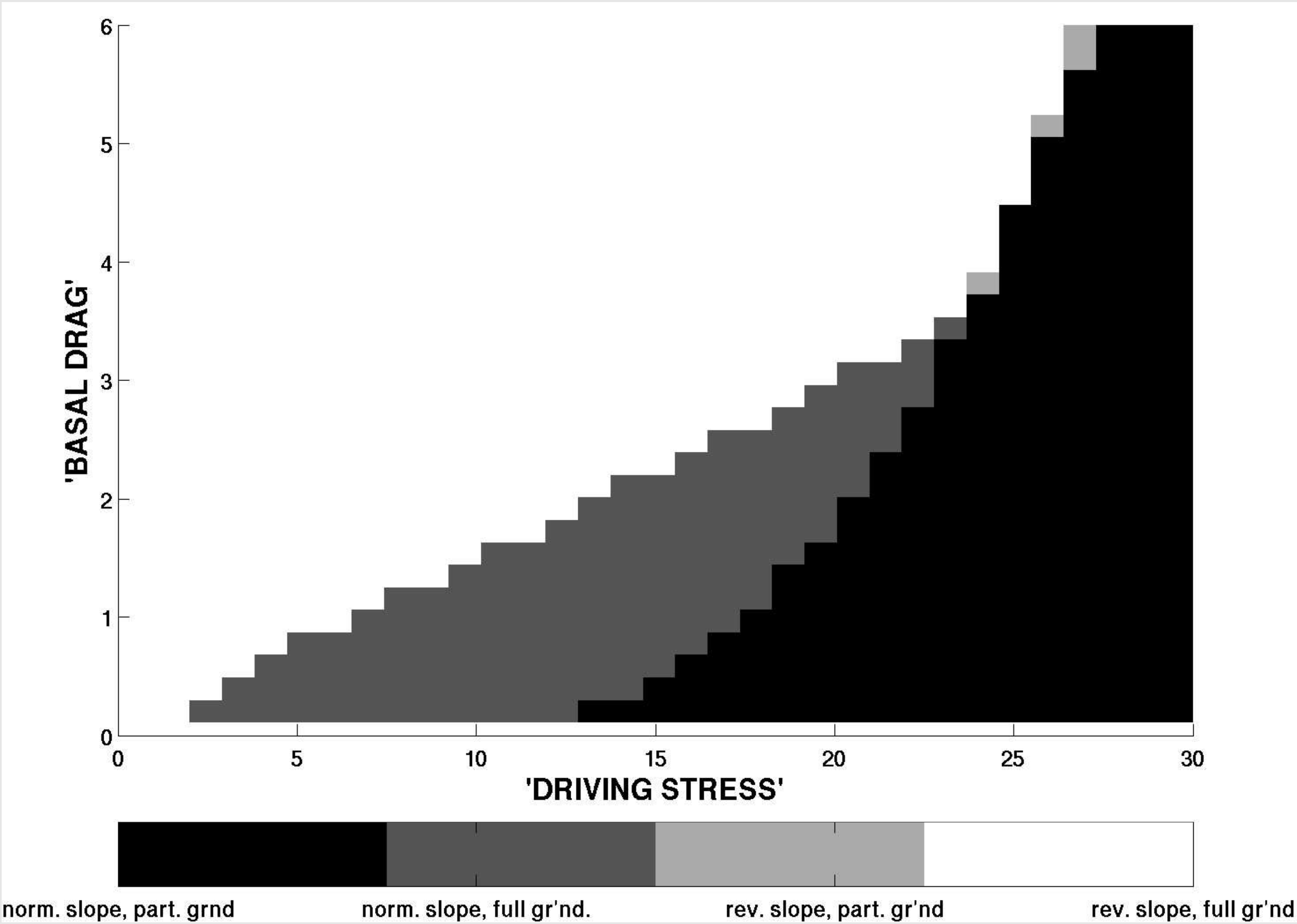


PENNSTATE

RESULTS: $h_0 = 1.1$, $f = 1$

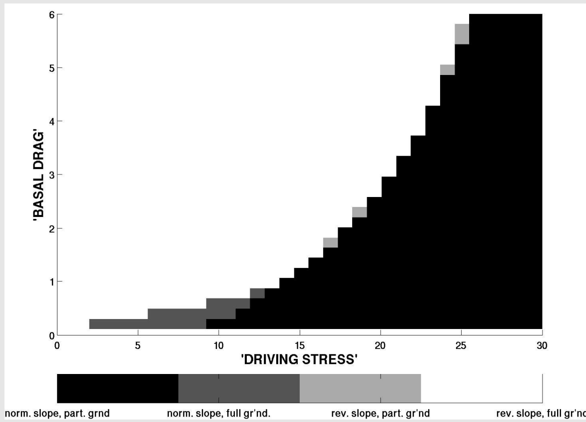


RESULTS: $h_0 = 1.1$, $f = 0.9$

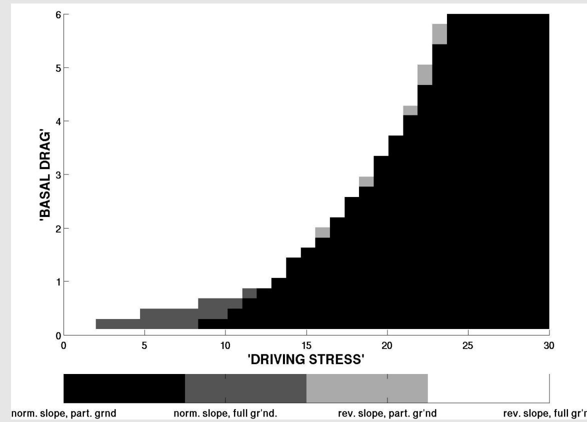


PENNSTATE

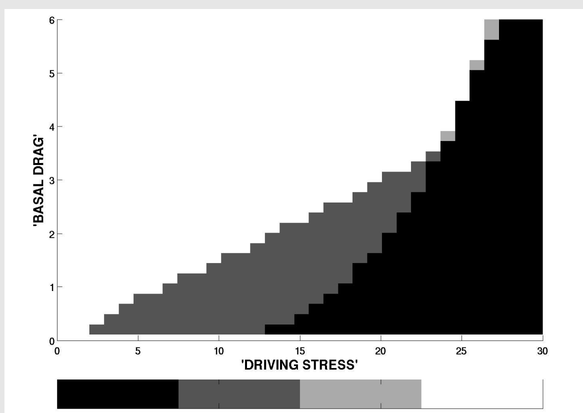
RESULTS SUMMARY



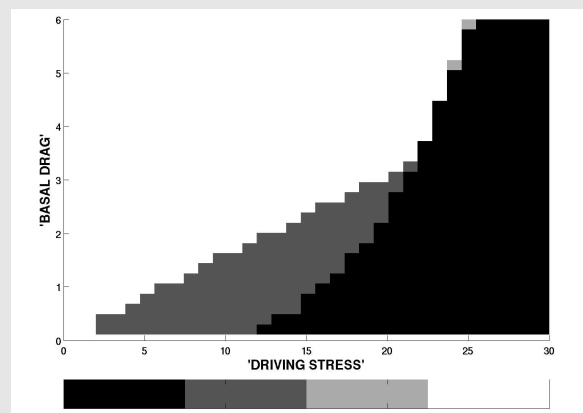
$h_0 = 1.01,\ f = 0.9$



$h_0 = 1.01,\ f = 1$



$h_0 = 1.1,\ f = 0.9$



$h_0 = 1.1,\ f = 1$

PENNSTATE

Initial Conclusions

- Slope reversals are found within the range of ’reasonable’ parameter values.
- Slope reversals tend to occupy the lower A_r, higher G portion of A_r, G space…
- Slope reversals are favored by a reduction in f or h_0.

Broader Conclusions

- Transition to a slope reversal is favored by
  - thinning
  - making the sill ’stickier’
  - increasing ice-shelf buttressing
- Grounding should cause slope reversals, and the slope reversal should dissapear later as the ice thickens.
- Outburst flooding is to be expected.

Future Work

- Topography - bump magnitude and shape
- Transient grounding scenarios
- Include a sealed water-filled basin

Acknowledgements

- Byron R. Parizek
- Matt Spencer
- All of the ice+climate members
- Ryan Elmore