Use of inverse methods to find basal velocities in ice stream onset areas

Martin Truffer David Maxwell

Geophysical Institute and Dept. of Math. and Stat.
University of Alaska Fairbanks

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Outline

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Ice Stream onset areas

- Transition from ice sheet to ice shelf flow

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- Yet another free boundary problem in glaciology (–> Schoof)

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- This might be important when modeling the evolution of onset area

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- This is not directly possible for the distribution of basal velocities
- Hence the need for inverse methods: derive basal velocities from observations at the surface (or from remote sensing)

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- Incidentally this is the same nonlinear Poisson equation that describes full order Stokes flow of a nonlinear fluid through a glacier's cross section with no out-of-plane gradients.

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- This problem is ill-posed and generally no solution is guaranteed
- Inverse methods seek an approximate solution

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- Repeat and stop when the solution has sufficiently converged.

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- This has been applied for the problem of isothermal flow through a glacier cross section (Maxwell et al., submitted)

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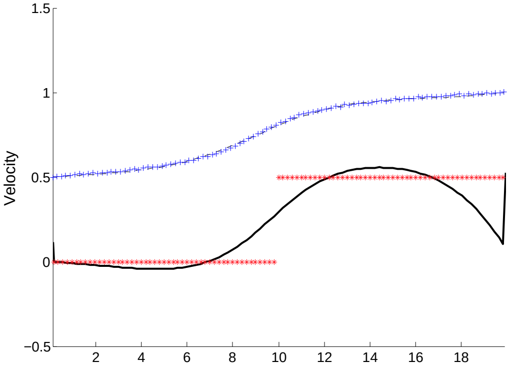
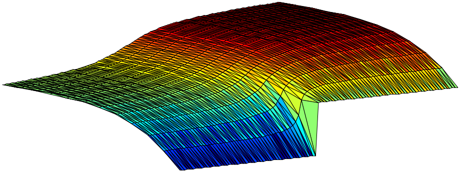
- We extract the calculated surface velocity and add noise
- We feed this data set to the inverse model and compare the resulting basal velocity to the one originally prescribed

Frozen - sliding transition

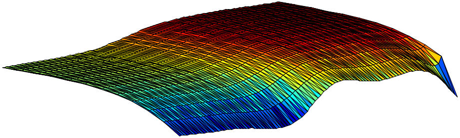


Frozen - sliding transition

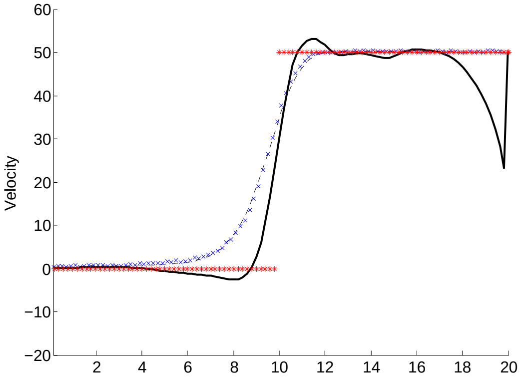
Sheet−Sheet forward



Sheet−Sheet inverse



Sheet to Shelf flow transition



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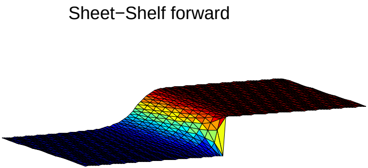

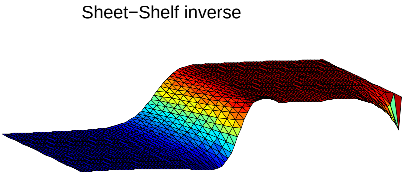

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- There are some fundamental limits on the spatial scale at which this can be done (one ice thickness)
- The method as shown assumes that rheological parameters are well known. That is a huge assumption.

Acknowledgements

- NSF for funding

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- You for listening