Boundary conditions for a full-momentum solver: 1) The dilemma of sliding and 2) how do we do embedded models?

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The holy grail of ice sheet modeling is the full-momentum solver. In principle, conservation of momentum, coupled with a flow law, can provide a differential equation solvable for velocities at every point within the ice sheet volume. The shallow-ice approximation neglects all but the basal drag and is useful for slow-moving inland ice. The Morland-MacAyeal equations neglect all but the longitudinal stresses and are useful for ice shelves and perhaps, in limited circumstances, ice streams. Both of these approximations take advantage of the different scales of the horizontal versus the vertical dimensions of the ice sheet, and involve an integration and removal of the vertical coordinate. Both of these approximations have severe limitations, especially in the dynamically critical ice streams that drain most of the mass out of Antarctica. The key interaction of shelf and inland ice, though the ice stream, cannot be adequately captured with either of these "end-member" approximations.

While very computationally intensive, a full-momentum solver that neglects no stresses and makes no assumptions or vertical integrations should give us the best and most accurate model for ice streams. Because of the computational requirements, such a model is not reasonable for a whole-ice sheet simulation, and hence we have pursued the embedded-grid approach, whereby a shallow-ice model is run for the whole ice sheet, and the full-momentum solver is applied only to a sub-region where ice stream dynamics are known to be important.

As such there are three different types of boundary conditions that must be specified, the top, the bottom, and the sides. The top is easy, a free-boundary is easily specified.

The sides and bottom are more difficult. On the sides we have a choice, either Dirichlet or Neumann boundary conditions. Dirichlet boundary conditions consist of specified boundary velocities (the unknowns, or degrees of freedom in the full-momentum solver), whereas Neumann boundary conditions are specified pressures or surface tractions (the source of momentum). From the shallow-ice model, we can provide both of these conditions, although the vertical variation in velocity is only of lower order. Pressures are not difficult to specify, however, conservation of angular momentum (net rotational torque should be zero), does require specification of some surface traction (the so-called " dynamical stresses"), and this can be problematic.

The bottom is most difficult, mainly due to the poorly understood nature of sliding (hard rock, deformable sediments, polythermal ice, basal water, etc.). A frozen bed is again easy, a simple Dirichlet boundary condition with velocities specified at zero. A completely uncoupled bottom is also easy, with simply a free boundary condition in the two horizontal dimensions. Of course in both of these cases the vertical velocity is specified to be zero.

Instead with Neumann boundary conditions, we can specify the basal traction. If this is specified to be equal to the driving stress (rho*g*h*alpha), we basically obtain the same solution that we get when we specify no sliding. We have tried specifying a given fraction of the driving stress, but this leads to unrealistic oscillations in the ice sheet profile. A uniform stress works well, but there is no indication that this is a reasonable assumption, nor does that deal well with the transition from inland to streaming to shelf. Both of these also require "yet-another-parameter," and hence are undesirable. A third approach involves a " deformable" basal layer, ie. a thin layer of elements, the order of meters thick, with much softer physical characteristics. With this approach, one can preserve the easy-to-implement Dirichlet boundary conditions of all basal velocities specified at zero, and yet obtain high sliding velocities, and plug-like flow, the dilemma being the requirement of a "parameter" (how soft and how thick is this layer?). Tuning such a model (and remember, this is how the parameters in most sliding laws are obtained, by tuning) would involve comparison of measured and modeled velocity fields in well-documented areas such as the Siple Coast, and soon the Amundsen Sector.